Typing & Static Analysis of Multi-Staged Programs

Kwangkeun Yi

Seoul National University, Korea

5/31/2011 @ UC Berkeley

(co-work with I. Kim, W. Choi, B. Aktemur, C. Calcagno, M. Tatsuda)

ROSAEC center
Research On Software Analysis for Error-free Computing
소프트웨어 무결점 연구센터 KOSEF ERC

About Us

We try to help reduce/eliminate errors in software.

- statically: before execution, before sell/embed
- automatically: against explosive sw size
- to find bugs or verify their absence

Our Activities

We published our works in:

- POPL('11, '06), TACAS('11), VMCAI('10, '11), ICSE('11), SAS, ISMM, OOPSLA, FSE, etc.
- TOPLAS, TCS, JFP, SP&E, Acta Informatica, etc.

A commercialization:

Sparrow
The Early Bird

Research areas: *static analysis, abstract interpretation, programming languge theory, type system, theorem proving, model checking, & whatever relevant*

Outline

1. Multi-staged Programming
2. Typing Multi-Staged Programs (POPL'06)
3. Static Analysis of Multi-Staged Programs (POPL'11)

Multi-Staged Programming (1/4)

program texts (code) as first class objects
"meta programming"

A general concept that subsumes

- web program's runtime code generation
- macros & templates
- Lisp's quasi-quotation
- partial evaluation

Common in JavaScript, Perl, PHP, Python, Lisp/Scheme, C's macros, C++ & Haskell's templates, C#, etc.

Multi-Staged Programming (2/4)

- divides a computation into stages
- program at stage 0: conventional program
- program at stage $n+1$: code as data at stage n

Stage	Computation	Value
0	usual + code + run	usual + code
> 0	code substitution	code

In examples, we will use Lisp-style staging constructs + only 2 stages

$$
\begin{array}{rcll}
e & ::= & \cdots & \\
 & | & \text{`}e & \text{code as data} \\
 & | & ,e & \text{code substitution} \\
 & | & \texttt{run}\ e & \text{execute code}
\end{array}
$$

- code as a value: `` `(1+1) ``
- code composition: `` let y = `(x+1) in '(λx.,y) ``
- code execution: `run '(1+1)`

Multi-Staged Programming (4/4)

Specializer/Partial evaluator

```
power(x,n) = if n=0 then 1 else x * power(x,n-1)
```

v.s. `power(x,3) = x*x*x`

prepared as

```
let spower(n) = if n=0 then ‘1 else ‘(x*,(spower (n-1)))
let fastpower = ‘(λx.,(spower input))
in (run fastpower) 2
```

Practice of Multi-Staged Programming

- open code

$$`(x+1)$$

- intentional variable-capturing substitution

$$\texttt{let y = `(x+1) in `(}\lambda\texttt{x.,y)}$$

- capture-avoiding substitution

$$\texttt{let y = `(x+1) in `(}\lambda^{*}\texttt{x.,y + x)}$$

- imperative operations with open code

$$\texttt{cell := `(x+1);} \cdots \texttt{cell := `(y 1);}$$

Challenge I

A static type system that supports the practice.

- type safety and
- the expressiveness of fully-fledged multi-staging operators

Previous type systems support only part of the practice.

Challenge II

A general, static analysis method for multi-staged programs.

The objects (program texts) to analyze

- are dynamic entities, which
- are only estimated by static analysis

Conventional analysis may fail to handle "`run` e"

No general static analysis method before.

ROSAEC center
Research On Software Analysis for Error-free Computing
소프트웨어 무결점 연구센터 KOSEF ERC

Part I: Our Answer I

A type system for (ML + Lisp's quasi-quote system)

- supports all in multi-staged programming practice
  - open code: `` `(x+1) ``
  - unrestricted imperative operations with open code
  - intentional var-capturing substitution at stages > 0
  - capture-avoiding substitution at stages > 0
- conservative extension of ML's let-polymorphism
- principal type inference algorithm

A Let-Polymorphic Modal Type System for Lisp-style Multi-Staged Programming [Kim, Yi, Calcagno: POPL'06]

ROSAEC center
Research On Software Analysis for Error-free Computing
소프트웨어 무결점 연구센터 KOSEF ERC

Ideas

- code's type: parameterized by its expected context

$$\Box(\Gamma \triangleright \mathit{int})$$

- view the type environment Γ as a record type

$$\Gamma = \{x : \mathit{int},\ y : \mathit{int} \rightarrow \mathit{int}, \cdots\}$$

- stages by the stack of type environments (modal logic S4)

$$\Gamma_0 \cdots \Gamma_n \vdash e : A$$

- with "due" restrictions
  - let-polymorphism for syntactic values
  - monomorphic Γ in code type $\Box(\Gamma \triangleright \mathit{int})$
  - monomorphic store types

Natural ideas worked.

Simple Type System

$$\textit{Type} \quad A, B \quad ::= \quad \iota \mid A \to B \mid \Box(\Gamma \triangleright A)$$

code type

$$\texttt{`(x+1)}: \ \Box(\{x : \texttt{int}, \cdots\} \triangleright \texttt{int})$$

typing judgment

$$\Gamma_0 \cdots \Gamma_n \vdash e : A$$

(TSBOX) $$\frac{\Gamma_0 \cdots \Gamma_n \Gamma \vdash e : A}{\Gamma_0 \cdots \Gamma_n \vdash \texttt{box}\, e : \Box(\Gamma \triangleright A)}$$

(TSUNBOX) $$\frac{\Gamma_0 \cdots \Gamma_n \vdash e : \Box(\Gamma_{n+k} \triangleright A)}{\Gamma_0 \cdots \Gamma_n \cdots \Gamma_{n+k} \vdash \texttt{unbox}_k e : A}$$

(TSEVAL) $$\frac{\Gamma_0 \cdots \Gamma_n \vdash e : \Box(\varnothing \triangleright A)}{\Gamma_0 \cdots \Gamma_n \vdash \texttt{run}\ \ e : A}$$ (for alpha-equiv. at stage 0)

Polymorphic Type System (1/2)

A combination of

- ML's let-polymorphism
  - syntactic value restriction + multi-staged "$\text{expansive}^n(e)$"
  - $\text{expansive}^n(e) = \mathit{False}$

    $\Longrightarrow$ e never expands the store during its eval. at $\forall$stages$\leq n$

    e.g.) ${}^{\backprime}(\lambda x.,e)$: can be expansive

    ${}^{\backprime}(\lambda x.\texttt{run}\, y)$: unexpansive

- Rémy's record types [Rémy 1993]
  - type environments as record types with field addition
  - record subtyping + record polymorphism

Polymorphic Type System (2/2)

- `if` e `then ‘(x+1) else ‘1`: $\boxed{\Box(\{x : int\}\rho \triangleright int)}$
  - then-branch: $\Box(\{x : int\}\rho' \triangleright int)$
  - else-branch: $\Box(\rho'' \triangleright int)$

- `let x = ‘y in ‘(,x + w); ‘((,x 1) + z)`

  $\boxed{\texttt{x}: \forall\alpha\forall\rho.\Box(\{y : \alpha\}\rho \triangleright \alpha)}$
  - first x: $\Box(\{y : int,\ w : int\}\rho' \triangleright int)$
  - second x: $\Box(\{y : int \to int,\ z : int\}\rho'' \triangleright int \to int)$

Research On Software Analysis for Error-free Computing
소프트웨어 무결점 연구센터 KOSEF ERC

Type Inference Algorithm

- Unification:
  - Rémy's unification for record type Γ
  - usual unification for new type terms such as $\Box(\Gamma \triangleright A)$ and $A\,\mathsf{ref}$
- Sound and complete principal type inference:
  - the same structure as top-down version $\mathcal{M}$ [Lee and Yi 1998] of the $\mathcal{W}$
  - usual on-the-fly instantiation and unification

Part I: Message

Staged programming "practice" has a sound static type system.

Part II: Challenge (rephrase)

A general, static analysis method for multi-staged programs.

The objects (program texts) to analyze

- are dynamic entities, which
- are only *estimated* by static analysis

How to analyze "`run` e", the execution of estimated program texts?

[Choi, Aktemur, Yi, Tatsuda: POPL'11] Static Analysis of Multi-Staged Programs via Unstaging Translation

Problem in Static Anaysis of Staged Programs

```
x := `0;
repeat x := `(,x + 2)
until cond;
run x
```

- The set of possible code for x:

$$\{\text{`}0, \text{`}(0+2), \text{`}(0+2+2), \cdots\}.$$

  must first be finitely approximated, e.g., by a grammar:

$$S \to 0 \mid S+2.$$

- analyzing "run x" needs code, not the grammar.

Our Solution

a detour: translate, analyze, and project.

1. unstaging translation
   - proof of semantic-preserving
2. conventional static analysis
   - can apply all existing static analysis techniques
3. cast the result back in terms of original staged programs
   - a sound condition for the projection
   - i.e., to be aligned with the correspondence induced by the translation.

Research On Software Analysis for Error-free Computing
소프트웨어 무결점 연구센터 KOSEF ERC

Translation Languages

Staged source		Unstaged target
$e ::= \lambda x.e \mid e\ e \mid x \mid \text{`}e \mid ,e \mid \texttt{run}\ e$	$\longmapsto$	$e ::= \lambda x.e \mid e\ e \mid x \mid \{\} \mid e\{\texttt{x}{=}e\} \mid e \cdot \texttt{x}$

Staged source

$$
\begin{array}{rcl}
e & ::= & \lambda x.e \\
  & \mid & e\ e \\
  & \mid & x \\
  & \mid & \text{`}\, e \\
  & \mid & ,\, e \\
  & \mid & \texttt{run}\ e
\end{array}
$$

$\longmapsto$

Unstaged target

$$
\begin{array}{rcl}
e & ::= & \lambda x.e \\
  & \mid & e\ e \\
  & \mid & x \\
  & \mid & \{\} \\
  & \mid & e\{\texttt{x=}e\} \\
  & \mid & e \cdot \texttt{x}
\end{array}
$$

Translation Ideas (1/2)

- code into env-taking function:

$$\texttt{`(1+1)} \longmapsto \lambda\rho.\texttt{1+1}$$

- free variable in a code into record lookup:

$$\texttt{`(}x\texttt{+1)} \longmapsto \lambda\rho.(\rho\cdot\texttt{x}) + 1$$

- `run` expression into an application:

$$\texttt{run `(1+1)} \longmapsto (\lambda\rho.\texttt{1+1})\{\}$$

- code composition into an app. whose actual param. is for the code-to-be-plugged expr.:

$$\texttt{`(,}y\texttt{ + 2)} \longmapsto (\lambda h\texttt{.}(\lambda\rho\texttt{.}(h\ \ \rho)\texttt{+2}))\ \ y$$

- variable capturing into record passing+lookup:

$$\texttt{'(}\lambda x\texttt{.,(`(}x\texttt{+1)))} \longmapsto \lambda\rho_1\lambda x\texttt{.((}\lambda\rho_2\texttt{.(}\rho_2\cdot\texttt{x)+1)}\ \ \texttt{(}\rho_1\{\texttt{x} = x\}\texttt{))}$$

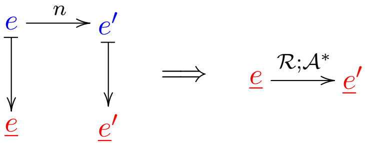
Translation Example

$$
\begin{array}{ll}
x \ \texttt{:=} \ \text{`}\texttt{0}\texttt{;} & \\
\texttt{repeat} & \\
\quad x := \text{`}(,x + 2) & \longmapsto \\
\texttt{until} \ cond\texttt{;} & \\
\texttt{run} \ x &
\end{array}
\qquad
\begin{array}{l}
x \ \texttt{:=} \ \lambda\rho\texttt{.0;} \\
\texttt{repeat} \\
\quad x \ \texttt{:=} \ \texttt{(}\lambda h\texttt{.(}\lambda\rho\texttt{.(}h \ \ \rho\texttt{)+2))} \ x \\
\texttt{until} \ cond\texttt{;} \\
x \ \{\}
\end{array}
$$

Simulation

Theorem

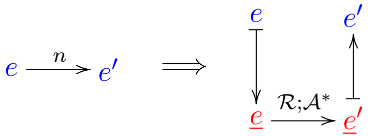
(Simulation) Let e *be a stage-*n $\lambda_{\mathcal{S}}$ *expression with no free variables such that* $e \xrightarrow{n} e'$*. Let* $R \vdash e \mapsto (\underline{e}, K)$ *and* $R \vdash e' \mapsto (\underline{e}', K')$*. Then* $K(\underline{e}) \xrightarrow{\mathcal{R};\mathcal{A}^*} K'(\underline{e}')$*.*



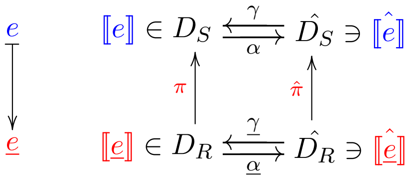
Inversion

Theorem

(Inversion) Let e *be a* $\lambda_{\mathcal{S}}$ *expression and* R *be an environment stack. If* $R \vdash e \mapsto (\underline{e}, K)$, *then* $H \vdash \underline{e} \mapsto e$ *for any* H *such that* $\overline{K} \subseteq H$.



Analysis and Projection



Theorem

(Projection) Let e *and* $\underline{e}$ *be, respectively, a staged program and its translated unstaged version. If* $[\![e]\!] \sqsubseteq \pi[\![\underline{e}]\!]$ *and* $\alpha \circ \pi \circ \underline{\gamma} \sqsubseteq \hat{\pi}$ *then* $\alpha[\![e]\!] \sqsubseteq \hat{\pi}[\![\hat{\underline{e}}]\!]$.

Example (1/5): $[\![e]\!]$ staged collecting semantics

$$
\begin{array}{l}
x \;\texttt{:=}\; \texttt{`0;} \\
\texttt{repeat} \\
\quad x := \text{`}(,x + 2) \\
\texttt{until}\;\; cond; \\
\texttt{run}\;\; x
\end{array}
$$

Collecting semantics $[\![e]\!] =$

$$
\begin{array}{rll}
x & \text{has} & \{\texttt{`0}, \texttt{`(0+2)}, \texttt{`(0+2+2)}, \cdots\} \\
\texttt{run}\; x & \text{has} & \{0, 2, 4, 6, \cdots\}
\end{array}
$$

Research On Software Analysis for Error-free Computing
소프트웨어 무결점 연구센터 KOSEF ERC

Example (2/5): $[\![e]\!]$ unstaged collecting semantics

$$
\begin{array}{l}
x \ := \ \lambda\rho_1.0;\\
\texttt{repeat}\\
\quad x \ := \ (\lambda h.(\lambda\rho_2.(h\ \ \rho_2)+2))\ \ x\\
\texttt{until}\ \ cond;\\
x\ \ \{\}
\end{array}
$$

Collecting semantics $[\![e]\!] =$

$$
\begin{array}{rll}
x, h & \text{has} & \{\langle\lambda\rho_1.0, \emptyset\rangle, \langle\lambda\rho_2.(h\ \ \rho_2)+2, \{h \mapsto \langle\lambda\rho_1.0\rangle\}\rangle, \cdots\}\\
\rho_1, \rho_2 & \text{has} & \{\}\\
x\ \ \{\} & \text{has} & \{0, 2, 4, 6, \cdots\}
\end{array}
$$

Example (3/5): π projection of collecting semantics

Collecting semantics are aligned:

$$[\![e]\!] \sqsubseteq \pi[\![\underline{e}]\!]$$

$$\begin{array}{lll} x, h & \text{has} & \{\langle \lambda\rho_1.\mathtt{0}, \emptyset\rangle, \\ & & \ \langle \lambda\rho_2.(h\ \ \rho_2)\mathtt{+2}, \\ & & \ \ \{h \mapsto \langle\lambda\rho_1.\mathtt{0}\rangle\}\rangle, \\ & & \ \cdots\} \\ \rho_1, \rho_2 & \text{has} & \{\} \end{array} \quad \stackrel{\pi}{\longmapsto} \quad x \quad \text{has} \quad \{\text{`}\mathtt{0}, \text{`}\mathtt{(0+2)}, \text{`}\mathtt{(0+2+2)}, \cdots\}$$

- π = inverse translation + removing admin stuff
- intuition

$$\begin{array}{lcl} \text{``}\lambda\rho\text{''} & \stackrel{\pi}{\longmapsto} & \text{``code indexed as } \rho\text{''} \\ \text{``}h\ \rho\text{''} & \stackrel{\pi}{\longmapsto} & \text{``code-filling by } h\text{''} \end{array}$$

ROSAEC center
Research On Software Analysis for Error-free Computing
소프트웨어 무결점 연구센터 KOSEF ERC

```
x := λρ1.0;
repeat
    x := (λh.(λρ2.(h ρ2)+2)) x
until cond;
x {}
```

0-CFA analysis $[\![\hat{e}]\!]$ in set-constraint style

x	has	$\lambda\rho_1.0$			
x	has	$\lambda\rho_2.(h\ \rho_2)+2$	$(h\ \rho_1)$	has	$V_1 \to 0 \mid V_1+2$
h	has	$\lambda\rho_1.0$	$x\ \{\}$	has	$V_2 \to 0 \mid V_1+2$
h	has	$\lambda\rho_2.(h\ \rho_2)+2$			

Example (5/5): $\hat{\pi}$ projection of analysis

$$\begin{array}{rll} x & \text{has} & \lambda\rho_1.0 \\ x & \text{has} & \lambda\rho_2.(h\ \rho_2)+2 \\ h & \text{has} & \lambda\rho_1.0 \\ h & \text{has} & \lambda\rho_2.(h\ \rho_2)+2 \\ x\ \{\} & \text{has} & V \to 0 \mid V+2 \end{array} \quad \stackrel{\hat{\pi}}{\longmapsto} \quad \begin{array}{rll} x & \text{has} & S_1 \to \rho_1 \\ x & \text{has} & S_2 \to \rho_2(S) \\ & & S \to \rho_1 \mid \rho_2(S) \\ \texttt{run}\ x & \text{has} & V \to 0 \mid V+2 \end{array}$$

- intuition

$$\begin{array}{lcl} \text{``}\lambda\rho\text{''} & \stackrel{\hat{\pi}}{\longmapsto} & \text{``code indexed as } \rho\text{''} \\ \text{``}h\ \rho\text{''} & \stackrel{\hat{\pi}}{\longmapsto} & \text{``code-filling by } h\text{''} \end{array}$$

- $\hat{\pi}$ satisfies the safety condition: $\alpha \circ \pi \circ \underline{\gamma} \sqsubseteq \hat{\pi}$
- and was $[\![e]\!] \sqsubseteq \pi[\![\underline{e}]\!]$

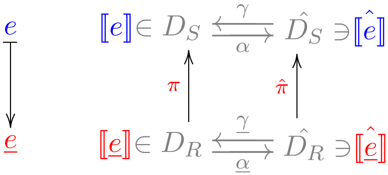
Hence, by the projection theoreom, correct:

$$\alpha[\![e]\!] \sqsubseteq \hat{\pi}[\![\hat{\underline{e}}]\!]$$

RSAEC center
Research On Software Analysis for Error-free Computing
소프트웨어 무결점 연구센터 KOSEF ERC

Part II: Conclusion

- semantic-preserving unstaging translation
- sound static analysis framework using the translation



unstaging + usual static analysis + projection are sufficient.

Things to Do

- extend to "string-based" (unstructured) multi-staged programming
- realistic static analyses: e.g. static malware detection
- program logic (e.g. separation logic) for multi-staging
- and any topic $\sim$ multi-staging