Typing & Static Analysis of Multi-Staged Programs

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About Us

We try to help reduce/eliminate errors in software.

- statically: before execution, before sell/embed
- automatically: against explosive sw size
- to find bugs or verify their absence

Our Activities

We published our works in:

- POPL('11, '06), TACAS('11), VMCAI('10, '11), ICSE('11), SAS, ISMM, OOPSLA, FSE, etc.
- TOPLAS, TCS, JFP, SP&E, Acta Informatica, etc.

A commercialization:

Sparrow
The Early Bird

Research areas: *static analysis, abstract interpretation, programming languge theory, type system, theorem proving, model checking, & whatever relevant*

Outline

1. Multi-Staged Programming
2. Typing Multi-Staged Programs (POPL'06)
3. Static Analysis of Multi-Staged Programs (POPL'11)

Multi-Staged Programming (1/4)

program texts (code) as first class objects
"meta programming"

A general concept that subsumes

- web program's runtime code generation
- macros & templates
- Lisp's quasi-quotation
- partial evaluation

Common in JavaScript, Perl, PHP, Python, Lisp/Scheme, C's macros, C++ & Haskell's templates, C#, etc.

Multi-Staged Programming (2/4)

- divides a computation into stages
- program at stage 0: conventional program
- program at stage $n+1$: code as data at stage n

Stage	Computation	Value
0	usual + code + run	usual + code
> 0	code substitution	code

In examples, we will use Lisp-style staging constructs + only 2 stages

$$
\begin{array}{rcll}
e & ::= & \cdots & \\
 & | & \text{`}e & \text{code as data} \\
 & | & ,e & \text{code substitution} \\
 & | & \texttt{run}\ e & \text{execute code}
\end{array}
$$

- code as a value: `` `(1+1) ``
- code composition: `` let y = `(x+1) in `(λx.,y) ``
- code execution: `` run `(1+1) ``

Multi-Staged Programming (4/4)

Specializer/Partial evaluator

```
power(x,n) = if n=0 then 1 else x * power(x,n-1)
```

v.s. `power(x,3) = x*x*x`

prepared as

```
let spower(n) = if n=0 then `1 else `(x*,(spower (n-1)))
let fastpower = `(λx.,(spower input))
in (run fastpower) 2
```

Practice of Multi-Staged Programming

- open code

$$\texttt{`(x+1)}$$

- intentional variable-capturing substitution

$$\texttt{let y = `(x+1) in `(}\lambda\texttt{x.,y)}$$

- capture-avoiding substitution

$$\texttt{let y = `(x+1) in `(}\lambda^*\texttt{x.,y + x)}$$

- imperative operations with open code

$$\texttt{cell := `(x+1);}\ \cdots\ \texttt{cell := `(y 1);}$$

Challenge I

A static type system that supports the practice.

- type safety and
- the expressivenss of fully-fledged multi-staging operators

Previous type systems support only part of the practice.

Challenge II

A general, static analysis method for multi-staged programs.

The objects (program texts) to analyze

- are dynamic entities, which
- are only estimated by static analysis

Conventional analysis may fail to handle "`run` e"

No general static analysis method before.

Part I: Our Answer I

A type system for (ML + Lisp's quasi-quote system)

- supports all in multi-staged programming practice
  - open code: `'(x+1)`
  - unrestricted imperative operations with open code
  - intentional var-capturing substitution at stages > 0
  - capture-avoiding substitution at stages > 0
- conservative extension of ML's let-polymorphism
- principal type inference algorithm

A Let-Polymorphic Modal Type System for Lisp-style Multi-Staged Programming [Kim, Yi, Calcagno: POPL'06]

Comparison

(1)	closed code and eval	(2)	open code
(3)	imperative operations	(4)	type inference
(5)	var-capturing subst.	(6)	capture-avoiding subst.
(7)	polymorphism		

Our system	+1	+2	+3	+4	+5	+6	+7
[Rhiger 2005]	+1	+2	+3	−4	+5	−6	−7
[Calcagno et al. 2004]	+1	+2	−3	+4	−5	+6	+7
[Ancona & Moggi 2004]	+1	+2	+3	−4	−5	+6	−7
[Taha & Nielson 2003]	+1	+2	−3	−4	−5	+6	+7
[Chen & Xi 2003]	+1	+2	+3	−4	+5	−6	+7
[Nanevsky & Pfenning 2002]	+1	+2	+3	−4	−5	+6	−7
MetaML/Ocaml[2000,2001]	+1	+2	−3	+4	−5	+6	+7
[Davies 1996]	−1	+2	−3	−4	−5	+6	−7
[Davies & Pfenning 1996,2001]	+1	−2	+3	+4	−5	+6	−7

Ideas

- code's type: parameterized by its expected context

$$\Box(\Gamma \triangleright \mathit{int})$$

- view the type environment Γ as a record type

$$\Gamma = \{x : \mathit{int},\ y : \mathit{int} \to \mathit{int}, \cdots\}$$

- stages by the stack of type environments (modal logic S4)

$$\Gamma_0 \cdots \Gamma_n \vdash e : A$$

- with "due" restrictions
  - let-polymorphism for syntactic values
  - monomorphic Γ in code type $\Box(\Gamma \triangleright \mathit{int})$
  - monomorphic store types

Natural ideas worked.

$$\textit{Type} \quad A, B \quad ::= \quad \iota \mid A \to B \mid \Box(\Gamma \triangleright A)$$

code type

$$\text{`(x+1)}: \ \Box(\{x : \texttt{int}, \cdots\} \triangleright \texttt{int})$$

typing judgment

$$\Gamma_0 \cdots \Gamma_n \vdash e : A$$

(TSBOX) $$\frac{\Gamma_0 \cdots \Gamma_n \Gamma \vdash e : A}{\Gamma_0 \cdots \Gamma_n \vdash \texttt{box}\ e : \Box(\Gamma \triangleright A)}$$

(TSUNBOX) $$\frac{\Gamma_0 \cdots \Gamma_n \vdash e : \Box(\Gamma_{n+k} \triangleright A)}{\Gamma_0 \cdots \Gamma_n \cdots \Gamma_{n+k} \vdash \texttt{unbox}_k e : A}$$

(TSEVAL) $$\frac{\Gamma_0 \cdots \Gamma_n \vdash e : \Box(\emptyset \triangleright A)}{\Gamma_0 \cdots \Gamma_n \vdash \texttt{run}\ \ e : A}$$ (for alpha-equiv. at stage 0)

Polymorphic Type System (1/2)

A combination of

- ML's let-polymorphism
  - syntactic value restriction + multi-staged "$\text{expansive}^n(e)$"
  - $\text{expansive}^n(e) = $ *False*

    $\Longrightarrow$ e never expands the store during its eval. at $\forall$stages$\leq n$

$$\begin{array}{llll} \text{e.g.)} & \text{`}(\lambda x.,e) & : & \text{can be expansive} \\ & \text{`}(\lambda x.\texttt{run}\,y) & : & \text{unexpansive} \end{array}$$

- Rémy's record types [Rémy 1993]
  - type environments as record types with field addition
  - record subtyping + record polymorphism

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Polymorphic Type System (2/2)

- `if` e `then ‘(x+1) else ‘1`: $\boxed{\Box(\{x : int\}\rho \triangleright int)}$
  - then-branch: $\Box(\{x : int\}\rho' \triangleright int)$
  - else-branch: $\Box(\rho'' \triangleright int)$

- `let x = ‘y in ‘(,x + w); ‘((,x 1) + z)`
  $\boxed{\texttt{x}: \forall\alpha\forall\rho.\Box(\{y : \alpha\}\rho \triangleright \alpha)}$
  - first x: $\Box(\{y : int,\ w : int\}\rho' \triangleright int)$
  - second x: $\Box(\{y : int \to int,\ z : int\}\rho'' \triangleright int \to int)$

Type Inference Algorithm

- Unification:
  - Rémy's unification for record type Γ
  - usual unification for new type terms such as $\Box(\Gamma \triangleright A)$ and $A\,\mathrm{ref}$
- Sound and complete principal type inference:
  - the same structure as top-down version $\mathcal{M}$ [Lee and Yi 1998] of the $\mathcal{W}$
  - usual on-the-fly instantiation and unification

Part I: Message

Staged programming "practice" has a sound static type system.

Part II: Challenge (rephrase)

A general, static analysis method for multi-staged programs.

The objects (program texts) to analyze

- are dynamic entities, which
- are only estimated by static analysis

Conventional analysis may fail to handle "`run` e"

- how to analyze the run of estimated program texts?

[Choi, Aktemur, Yi, Tatsuda: POPL'11] Static Analysis of Multi-Staged Programs via Unstaging Translation

Problem in Static Anaysis of Staged Programs

```
x := '0;
repeat x := '(,x + 2)
until cond;
run x
```

- The set of possible code for x:

$$\{\texttt{`0}, \texttt{`(0+2)}, \texttt{`(0+2+2)}, \cdots\}.$$

  must first be finitely approximated, e.g., by a grammar:

$$S \to \texttt{0} \mid S\texttt{+2}.$$

- analyzing "run x" needs code, not the grammar.

Our Solution

a detour: translate, analyze, and project.

step 1. unstaging translation

- proof of semantic-preserving

step 2. conventional static analysis

- can apply all existing static analysis techniques

step 3. cast the result back in terms of original staged programs

- a sound condition for the projection
- i.e., to be aligned with the correspondence induced by the translation.

Translation Languages

Staged source		Unstaged target
$e ::= \lambda x.e \mid e\ e \mid x \mid \text{`}e \mid ,e \mid \mathtt{run}\ e$	$\longmapsto$	$e ::= \lambda x.e \mid e\ e \mid x \mid \{\} \mid e\{\mathtt{x}{=}e\} \mid e \cdot \mathtt{x}$

Translation Ideas (1/2)

- code into env-taking function:

$$\texttt{`(1+1)} \longmapsto \lambda\rho.\texttt{1+1}$$

- free variable in a code into record lookup:

$$\texttt{`(}x\texttt{+1)} \longmapsto \lambda\rho.(\rho\cdot\texttt{x}) + 1$$

- `run` expression into an application:

$$\texttt{run `(1+1)} \longmapsto (\lambda\rho.\texttt{1+1})\{\}$$

Translation Ideas (2/2)

- code composition into an app. whose actual param. is for the code-to-be-plugged expr.:

$$\texttt{`(,}y\ \texttt{+ 2)} \longmapsto \texttt{(}\lambda h\texttt{.(}\lambda\rho\texttt{.(}h\ \ \rho\texttt{)+2))}\ \ y$$

- variable capturing into record passing+lookup:

$$\texttt{'(}\lambda x\texttt{.,(`(}x\texttt{+1)))} \longmapsto \lambda\rho_1\lambda x\texttt{.((}\lambda\rho_2\texttt{.(}\rho_2{\cdot}\texttt{x)+1)}\ \ \texttt{(}\rho_1\{\texttt{x} = x\}\texttt{))}$$

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Translation Example

x := ‘0;
repeat
  x := ‘(,x + 2)
until $cond$;
run x

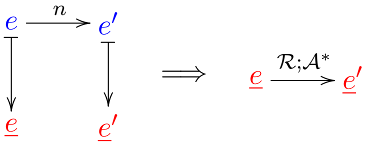
$\longmapsto$

x := $\lambda\rho$.0;
repeat
  x := (λh.($\lambda\rho$.(h ρ)+2)) x
until $cond$;
x {}

Simulation

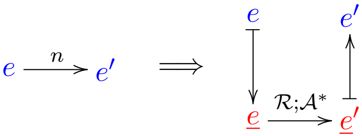
Theorem

(Simulation) Let e be a stage-n $\lambda_{\mathcal{S}}$ expression with no free variables such that $e \xrightarrow{n} e'$. Let $R \vdash e \mapsto (\underline{e}, K)$ and $R \vdash e' \mapsto (\underline{e}', K')$. Then $K(\underline{e}) \xrightarrow{\mathcal{R};\mathcal{A}^*} K'(\underline{e}')$.



Theorem

(Inversion) Let e be a $\lambda_{\mathcal{S}}$ expression and R be an environment stack. If $R \vdash e \mapsto (\underline{e}, K)$, then $H \vdash \underline{e} \mapsto e$ for any H such that $\overline{K} \subseteq H$.



Analysis and Projection

$$
\begin{array}{ccccccc}
e & & [\![e]\!] \in D_S & \underset{\alpha}{\overset{\gamma}{\leftrightarrows}} & \hat{D}_S \ni \widehat{[\![e]\!]} \\
\big\downarrow & & \uparrow \pi & & \uparrow \hat{\pi} \\
\underline{e} & & [\![\underline{e}]\!] \in D_R & \underset{\underline{\alpha}}{\overset{\underline{\gamma}}{\leftrightarrows}} & \hat{D}_R \ni \widehat{[\![\underline{e}]\!]}
\end{array}
$$

Theorem

(Projection) Let e *and* $\underline{e}$ *be, respectively, a staged program and its translated unstaged version. If* $[\![e]\!] \sqsubseteq \pi[\![\underline{e}]\!]$ *and* $\alpha \circ \pi \circ \underline{\gamma} \sqsubseteq \hat{\pi}$ *then* $\alpha[\![e]\!] \sqsubseteq \hat{\pi}\widehat{[\![\underline{e}]\!]}$.

Example (1/5): $[\![e]\!]$ staged collecting semantics

$$
\begin{array}{l}
x \;\texttt{:=}\; \texttt{`0;} \\
\texttt{repeat} \\
\quad x := \text{`}(,x + 2) \\
\texttt{until}\;\; cond\texttt{;} \\
\texttt{run}\;\; x
\end{array}
$$

Collecting semantics $[\![e]\!] =$

$$
\begin{array}{rcl}
x & \text{has} & \{\texttt{`0}, \texttt{`(0+2)}, \texttt{`(0+2+2)}, \cdots\} \\
\texttt{run}\;\, x & \text{has} & \{0, 2, 4, 6, \cdots\}
\end{array}
$$

$$
\begin{array}{l}
x \;\texttt{:=}\; \lambda\rho_1\texttt{.0;} \\
\texttt{repeat} \\
\quad x \;\texttt{:=}\; \texttt{(}\lambda h\texttt{.(}\lambda\rho_2\texttt{.(}h\;\;\rho_2\texttt{)+2))}\;\; x \\
\texttt{until}\;\; cond\texttt{;} \\
x\;\; \{\}
\end{array}
$$

Collecting semantics $[\![\underline{e}]\!] =$

$$
\begin{array}{rll}
x, h & \text{has} & \{\langle\lambda\rho_1.0, \emptyset\rangle, \langle\lambda\rho_2.(h\;\;\rho_2)+2, \{h \mapsto \langle\lambda\rho_1.0\rangle\}\rangle, \cdots\} \\
\rho_1, \rho_2 & \text{has} & \{\} \\
x\;\;\{\} & \text{has} & \{0, 2, 4, 6, \cdots\}
\end{array}
$$

Example (3/5): π projection of collecting semantics

Collecting semantics are aligned:

$$[\![e]\!] \sqsubseteq \pi[\![\underline{e}]\!]$$

$$\begin{array}{lll} x, h & \text{has} & \{\langle \lambda\rho_1.\mathtt{0}, \emptyset\rangle, \\ & & \ \langle \lambda\rho_2.\mathtt{(}h\ \ \rho_2\mathtt{)+2}, \\ & & \ \ \{h \mapsto \langle\lambda\rho_1.\mathtt{0}\rangle\}\rangle, \\ & & \cdots\} \\ \rho_1, \rho_2 & \text{has} & \{\} \end{array} \quad \stackrel{\pi}{\longmapsto} \quad x \quad \text{has} \quad \begin{array}{l} \{\text{`}\mathtt{0}, \text{`}\mathtt{(0+2)}, \\ \ \ \text{`}\mathtt{(0+2+2)}, \cdots\} \end{array}$$

- π = inverse translation + removing admin stuff
- intuition

$$\begin{array}{ccc} \text{“}\lambda\rho\text{”} & \stackrel{\pi}{\longmapsto} & \text{“code indexed as } \rho\text{”} \\ \text{“}h\ \rho\text{”} & \stackrel{\pi}{\longmapsto} & \text{“code-filling by } h\text{”} \end{array}$$

Example (4/5): $\hat{[\![e]\!]}$ unstaged conventional analysis

$$
\begin{array}{l}
x \ \texttt{:=}\ \lambda\rho_1\texttt{.0;} \\
\texttt{repeat} \\
\quad x \ \texttt{:=}\ (\lambda h.(\lambda\rho_2.(h\ \rho_2)\texttt{+2}))\ x \\
\texttt{until}\ \mathit{cond}; \\
x\ \{\}
\end{array}
$$

0-CFA analysis $\hat{[\![e]\!]}$ in set-constraint style

$$
\begin{array}{lll}
x & \text{has} & \lambda\rho_1.0 \\
x & \text{has} & \lambda\rho_2.(h\ \rho_2)+2 \\
h & \text{has} & \lambda\rho_1.0 \\
h & \text{has} & \lambda\rho_2.(h\ \rho_2)+2
\end{array}
\qquad
\begin{array}{rll}
(h\ \rho_1) & \text{has} & V_1 \to 0 \mid V_1+2 \\
x\ \{\} & \text{has} & V_2 \to 0 \mid V_1+2
\end{array}
$$

Example (5/5): $\hat{\pi}$ projection of analysis

$$\begin{array}{rll} x & \text{has} & \lambda\rho_1.0 \\ x & \text{has} & \lambda\rho_2.(h\ \rho_2)+2 \\ h & \text{has} & \lambda\rho_1.0 \\ h & \text{has} & \lambda\rho_2.(h\ \rho_2)+2 \\ x\ \{\} & \text{has} & V \to 0 \mid V+2 \end{array} \quad \stackrel{\hat{\pi}}{\longmapsto} \quad \begin{array}{rll} x & \text{has} & S_1 \to \rho_1 \\ x & \text{has} & S_2 \to \rho_2(S) \\ & & S \to \rho_1 \mid \rho_2(S) \\ \texttt{run}\ x & \text{has} & V \to 0 \mid V+2 \end{array}$$

- intuition

$$\begin{array}{rcl} \text{“}\lambda\rho\text{''} & \stackrel{\hat{\pi}}{\longmapsto} & \text{“code indexed as } \rho\text{''} \\ \text{“}h\ \rho\text{''} & \stackrel{\hat{\pi}}{\longmapsto} & \text{“code-filling by } h\text{''} \end{array}$$

- $\hat{\pi}$ satisfies the safety condition: $\alpha \circ \pi \circ \underline{\gamma} \sqsubseteq \hat{\pi}$
- and was $[\![e]\!] \sqsubseteq \pi[\![\underline{e}]\!]$

Hence, by the projection theoreom, correct:

$$\alpha[\![e]\!] \sqsubseteq \hat{\pi}[\![\hat{\underline{e}}]\!]$$

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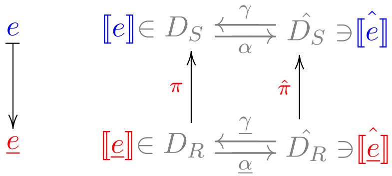
Part II: Conclusion

- semantic-preserving unstaging translation
- sound static analysis framework using the translation



unstaging + usual static analysis + projection are sufficient.

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Questions and Things to Do

- why not directly on staged programs?
  - typing worked because?
  - how, for more than typing?
- extend to "string-based" (unstructured) multi-staged programming
- program logic (e.g. separation logic) for multi-staging
- realistic static analyses
  - e.g. static Javascript malware detection