

Homework 1
Eng. Math. II: Logic in Computing
SNU 400.002, 2005 가을
이 광근
Due: 10/6(목), 16:00

Exercise 1 “Reading I” (50pts)

과목 홈페이지 [읽을거리]에 매달린 “19세기 논리학 21세기 계산학”, “Why Universities ...”, “Why Math”, “Mathematical Reasoning ...”을 읽고, 독후감을 제출한다. (5장 이내, 컴퓨터 출력)

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Exercise 2 “proof by induction” (20pts)

Set Exp consists only of e that are inductively defined as follows:

$$\begin{array}{lcl} e & \rightarrow & 0 \mid 2 \\ & | & x \quad \text{variable} \\ & | & e - e \\ & | & e \times e \end{array}$$

Prove

$\forall e \in Exp.$ if every variable in e has an even integer then e 's value is an even integer.

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Exercise 3 “proof construction” (20pts)

Recall the natural deduction rules of the two styles discussed in class. Show two proofs (one for each style) for each of the following sequents

- $\neg(p \vee q) \vdash \neg p \wedge \neg q$
- $\neg(p \wedge q) \vdash \neg p \vee \neg q$
- $p \vee (q \wedge r) \vdash (p \vee q) \wedge (p \vee r)$
- $p \wedge q, (p \wedge q \rightarrow r) \vdash (p \rightarrow r) \wedge (q \rightarrow r)$

□

Exercise 4 “proof = program” (30pts)

See for yourself the Curry-Howard isomorphism that proof is program and proof’s proposition is program’s type.

- Construct the proof tree of

$$(p \wedge q) \wedge (p \wedge q \rightarrow r) \rightarrow (p \rightarrow r) \wedge (q \rightarrow r)$$

using the sequent-style $(\Gamma \vdash \varphi)$ proof rules.

- Construct a program that corresponds to the above proof tree.

A program e is inductively defined as

$$e \rightarrow x \mid \lambda x.(e) \mid e(e) \mid (e, e) \mid e.1 \mid e.2$$

where “ x ” is a variable, “ $\lambda x.(e)$ ” is a function whose parameter is x and its body is e , “ $e_1(e_2)$ ” is function e_1 ’s application to e_2 , “ (e_1, e_2) ” is a tuple of e_1 and e_2 , and “ $e.i$ ” is the i -th component of tuple e . (읽을거리 중 “19세기 논리학 21세기 계산학” 참조)

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Exercise 5 (40pts) 교과서의 3.7 Exercises의 1번과 4번. □