4190.310 Programming Language
The K-- Language

1 Syntax

Expression e	$\rightarrow$	n	integer
	$\mid$	`true` $\mid$ `false`	boolean
	$\mid$	x	identifier
	$\mid$	e `+` e $\mid$ e `-` e $\mid$ e `*` e $\mid$ e `/` e	arithmetic expression
	$\mid$	e `<` e $\mid$ e `=` e $\mid$ `not` e	boolean expression
	$\mid$	x `:=` e	assignment
	$\mid$	e `;` e	sequence
	$\mid$	`if` e `then` e `else` e	branch
	$\mid$	`while` e `do` e	while loop
	$\mid$	`read` x	input
	$\mid$	`write` e	output
	$\mid$	`let` x `:=` e `in` e	local binding

1.1 Program

A program is an expression.

1.2 Identifiers

Alpha-numeric identifiers are `[a-zA-Z][a-zA-Z0-9_]`*. Identifiers are case sensitive: `z` and `Z` are different. The reserved words cannot be used as identifiers: `true false not if then else while do read write let in`

1.3 Numbers/Comments

Numbers are integers, optionally prefixed with `-`(for negative integer): `-?[0-9]`$^+$.

A comment is any character sequence within the comment block `(* *)`. The comment block can be nested.

1.4 Precedence/Associativity

In parsing K-- program text, the precedence of the K-- constructs in decreasing order is as follows. Symbols in the same set have identical precedence. Sym-

bols with subscript L (respectively R) are left (respectively right) associative. Symbols without subscript are nonassociative.

$$
\begin{array}{l}
\{\texttt{not}\}_R, \\
\{\texttt{*}, \texttt{/}\}_L, \\
\{\texttt{+}, \texttt{-}\}_L, \\
\{\texttt{=}, \texttt{<}\}_L, \\
\{\texttt{write}\}_R, \\
\{\texttt{:=}\}_R, \\
\{\texttt{else}\}, \\
\{\texttt{then}\}, \\
\{\texttt{do}\}, \\
\{\texttt{;}\}_L, \\
\{\texttt{in}\}
\end{array}
$$

For example, K-- program

```
x := e1; e2                 ⇒  (x := e1) ; e2
while e do e1; e2           ⇒  (while e do e1); e2
if e1 then e2 else e3; e4   ⇒  (if e1 then e2 else e3); e4
```

Rule of thumb: for your test programs, if your programs are hard to read (hence can be parsed not as you expected) then put parentheses around.

2 Semantics

$$
\begin{array}{llrll}
n & \in & \mathbb{Z} & & \text{integers} \\
b & \in & \mathbb{B} & & \text{booleans} \\
x, y & \in & \mathit{Id} & & \text{identifiers} \\
l & \in & \mathit{Addr} & & \text{addresses} \\
v & \in & \mathit{Val} & = & \mathbb{Z} + \mathbb{B} \\
\sigma & \in & \mathit{Env} & = & \mathit{Id} \xrightarrow{\text{fin}} \mathit{Addr} \\
M & \in & \mathit{Mem} & = & \mathit{Addr} \xrightarrow{\text{fin}} \mathit{Val}
\end{array}
$$

[Num] $$\frac{}{\sigma, M \vdash \texttt{n} \Downarrow n, M}$$

[True] $$\frac{}{\sigma, M \vdash \texttt{true} \Downarrow \mathit{true}, M}$$

[False] $$\frac{}{\sigma, M \vdash \texttt{false} \Downarrow \mathit{false}, M}$$

[Var] $$\frac{}{\sigma, M \vdash x \Downarrow M(\sigma(x)), M}$$

[Add] $$\frac{\sigma, M \vdash e_2 \Downarrow n_2, M_1 \qquad \sigma, M_1 \vdash e_1 \Downarrow n_1, M_2}{\sigma, M \vdash e_1 \texttt{ + } e_2 \Downarrow n_1 + n_2, M_2}$$

[Sub] $$\frac{\sigma, M \vdash e_2 \Downarrow n_2, M_1 \qquad \sigma, M_1 \vdash e_1 \Downarrow n_1, M_2}{\sigma, M \vdash e_1 \texttt{ - } e_2 \Downarrow n_1 - n_2, M_2}$$

[Mul] $$\frac{\sigma, M \vdash e_2 \Downarrow n_2, M_1 \qquad \sigma, M_1 \vdash e_1 \Downarrow n_1, M_2}{\sigma, M \vdash e_1 \texttt{ * } e_2 \Downarrow n_1 \times n_2, M_2}$$

[Div] $$\frac{\sigma, M \vdash e_2 \Downarrow n_2, M_1 \qquad \sigma, M_1 \vdash e_1 \Downarrow n_1, M_2}{\sigma, M \vdash e_1 \texttt{ / } e_2 \Downarrow n_1/n_2, M_2}$$

[EqT] $$\frac{\sigma, M \vdash e_2 \Downarrow v_2, M_1 \qquad \sigma, M_1 \vdash e_1 \Downarrow v_1, M_2 \qquad v_1 = v_2}{\sigma, M \vdash e_1 \texttt{ = } e_2 \Downarrow \mathit{true}, M_2}$$

[EqF] $$\frac{\sigma, M \vdash e_2 \Downarrow v_2, M_1 \qquad \sigma, M_1 \vdash e_1 \Downarrow v_1, M_2 \qquad v_1 \neq v_2}{\sigma, M \vdash e_1 \texttt{ = } e_2 \Downarrow \mathit{false}, M_2}$$

[Less] $$\frac{\sigma, M \vdash e_2 \Downarrow n_2, M_1 \qquad \sigma, M_1 \vdash e_1 \Downarrow n_1, M_2}{\sigma, M \vdash e_1 \texttt{ < } e_2 \Downarrow n_1 < n_2, M_2}$$

[Not] $$\frac{\sigma, M \vdash e \Downarrow b, M_1}{\sigma, M \vdash \texttt{not } e \Downarrow \mathit{not}\, b, M_1}$$

[Assign] $$\frac{\sigma, M \vdash e \Downarrow v, M_1}{\sigma, M \vdash x := e \Downarrow v, M_1\{\sigma(x) \mapsto v\}}$$

[Seq] $$\frac{\sigma, M \vdash e_1 \Downarrow v_1, M_1 \qquad \sigma, M_1 \vdash e_2 \Downarrow v_2, M_2}{\sigma, M \vdash e_1 \texttt{ ; } e_2 \Downarrow v_2, M_2}$$

[IfT] $$\frac{\sigma, M \vdash e \Downarrow \mathit{true}, M_1 \qquad \sigma, M_1 \vdash e_1 \Downarrow v, M_2}{\sigma, M \vdash \texttt{if } e \texttt{ then } e_1 \texttt{ else } e_2 \Downarrow v, M_2}$$

[IfF] $$\frac{\sigma, M \vdash e \Downarrow \mathit{false}, M_1 \qquad \sigma, M_1 \vdash e_2 \Downarrow v, M_2}{\sigma, M \vdash \texttt{if } e \texttt{ then } e_1 \texttt{ else } e_2 \Downarrow v, M_2}$$

[WhileT] $$\frac{\sigma, M \vdash e \Downarrow \mathit{true}, M_1 \quad \sigma, M_1 \vdash e_1 \Downarrow v_1, M_2 \qquad \sigma, M_2 \vdash \texttt{while } e \texttt{ do } e_1 \Downarrow v_2, M_3}{\sigma, M \vdash \texttt{while } e \texttt{ do } e_1 \Downarrow v_2, M_3}$$

[WhileF] $$\frac{\sigma, M \vdash e \Downarrow \mathit{false}, M_1}{\sigma, M \vdash \texttt{while } e \texttt{ do } e_1 \Downarrow \mathit{false}, M_1}$$

[Read] $$\frac{}{\sigma, M \vdash \texttt{read } x \Downarrow n, M\{\sigma(x) \mapsto n\}}$$

[Write] $$\frac{\sigma, M \vdash e \Downarrow n, M_1}{\sigma, M \vdash \texttt{write } e \Downarrow n, M_1}$$

[Let] $$\frac{\sigma, M \vdash e_1 \Downarrow v_1, M_1 \quad \ell \notin \mathit{dom}\, M_1 \qquad \sigma\{x \mapsto \ell\}, M_1\{\ell \mapsto v_1\} \vdash e_2 \Downarrow v_2, M_2}{\sigma, M \vdash \texttt{let } x := e_1 \texttt{ in } e_2 \Downarrow v_2, M_2}$$