4190.310 Programming Language
The K− Language

1 Syntax

Expression e	$\rightarrow$	n	integer
	$\mid$	`true` $\mid$ `false`	boolean
	$\mid$	x, f	identifier
	$\mid$	e `+` e $\mid$ e `-` e $\mid$ e `*` e $\mid$ e `/` e	arithmetic expression
	$\mid$	e `<` e $\mid$ e `=` e $\mid$ `not` e	boolean expression
	$\mid$	x `:=` e	assignment
	$\mid$	e `;` e	sequence
	$\mid$	`if` e `then` e `else` e	branch
	$\mid$	`while` e `do` e	while loop
	$\mid$	`read` x	input
	$\mid$	`write` e	output
	$\mid$	`let` x `:=` e `in` e	local variable binding
	$\mid$	`let proc` f `(` $x^*_,$ `)` `=` e `in` e	local procedure binding
	$\mid$	f `(` $e^*_,$ `)`	call by value
	$\mid$	f `<` $(x \mid e.x)^*_,$ `>`	call by reference
	$\mid$	`{` $(x$ `:=` $e)^*_,$ `}`	record
	$\mid$	$e.x$	record field
	$\mid$	$e.x$ `:=` e	record field assignment

Notation: α^*_β indicates zero or finite sequence of α's seperated by β. For example, $x^*_,$ indicates the empty string or "x" or "x`,` x" or "x`,` x`,` x" etc.

1.1 Program

A program is an expression.

1.2 Identifiers

Alpha-numeric identifiers are [`a-zA-Z`][`a-zA-Z0-9_`]*. Identifiers are case sensitive: `z` and `Z` are different. The reserved words cannot be used as identifiers: `true false not if then else while do read write let in proc`

1.3 Numbers/Comments

Numbers are integers, optionally prefixed with `-`(for negative integer): `-`?[`0-9`]$^+$.

A comment is any character sequence within the comment block `(* *)`. The comment block can be nested.

1.4 Precedence/Associativity

In parsing K-- program text, the precedence of the K-- constructs in decreasing order is as follows. Symbols in the same set have identical precedence. Symbols with subscript L (respectively R) are left (respectively right) associative. Symbols without subscript are nonassociative.

$$\begin{array}{l}
\{\texttt{.}\}_L, \\
\{\texttt{not}\}_R, \\
\{\texttt{*}, \texttt{/}\}_L, \\
\{\texttt{+}, \texttt{-}\}_L, \\
\{\texttt{=}, \texttt{<}\}_L, \\
\{\texttt{write}\}_R, \\
\{\texttt{:=}\}_R, \\
\{\texttt{else}\}, \\
\{\texttt{then}\}, \\
\{\texttt{do}\}, \\
\{\texttt{;}\}_L, \\
\{\texttt{in}\}
\end{array}$$

For example, K-- program

$$\begin{array}{lcl}
\texttt{x :=}\ e_1\ \texttt{;}\ e_2 & \Rightarrow & \texttt{(x :=}\ e_1\texttt{)}\ \texttt{;}\ e_2 \\
\texttt{while}\ e\ \texttt{do}\ e_1\ \texttt{;}\ e_2 & \Rightarrow & \texttt{(while}\ e\ \texttt{do}\ e_1\texttt{);}\ e_2 \\
\texttt{if}\ e_1\ \texttt{then}\ e_2\ \texttt{else}\ e_3\ \texttt{;}\ e_4 & \Rightarrow & \texttt{(if}\ e_1\ \texttt{then}\ e_2\ \texttt{else}\ e_3\texttt{);}\ e_4
\end{array}$$

Rule of thumb: for your test programs, if your programs are hard to read (hence can be parsed not as you expected) then put parentheses around.

2 Semantics

$$\begin{array}{rcrcl}
n & \in & \mathbb{Z} & & \text{integers} \\
b & \in & \mathbb{B} & & \text{booleans} \\
x, f & \in & \mathit{Id} & & \text{identifiers} \\
l & \in & \mathit{Addr} & & \text{addresses} \\
r & \in & \mathit{Record} & = & \mathit{Id} \xrightarrow{\text{fin}} \mathit{Addr} \\
v & \in & \mathit{Val} & = & \mathbb{Z} + \mathbb{B} + \mathit{Record} \\
\sigma & \in & \mathit{Env} & = & \mathit{Id} \xrightarrow{\text{fin}} \mathit{Addr} + \mathit{Procedure} \\
M & \in & \mathit{Mem} & = & \mathit{Addr} \xrightarrow{\text{fin}} \mathit{Val} \\
p, \langle X, e, \sigma \rangle & \in & \mathit{Procedure} & = & (\mathit{Id}\ \mathit{list}) \times \mathit{Expression} \times \mathit{Env}
\end{array}$$

[Num] $$\frac{}{\sigma, M \vdash \texttt{n} \Downarrow n, M}$$

[True] $$\frac{}{\sigma, M \vdash \texttt{true} \Downarrow \mathit{true}, M}$$

[False] $$\frac{}{\sigma, M \vdash \texttt{false} \Downarrow \mathit{false}, M}$$

[Var] $$\frac{}{\sigma, M \vdash x \Downarrow M(\sigma(x)), M}$$

[Add] $$\frac{\sigma, M \vdash e_2 \Downarrow n_2, M_1 \qquad \sigma, M_1 \vdash e_1 \Downarrow n_1, M_2}{\sigma, M \vdash e_1 \texttt{ + } e_2 \Downarrow n_1 + n_2, M_2}$$

[Sub] $$\frac{\sigma, M \vdash e_2 \Downarrow n_2, M_1 \qquad \sigma, M_1 \vdash e_1 \Downarrow n_1, M_2}{\sigma, M \vdash e_1 \texttt{ - } e_2 \Downarrow n_1 - n_2, M_2}$$

[Mul] $$\frac{\sigma, M \vdash e_2 \Downarrow n_2, M_1 \qquad \sigma, M_1 \vdash e_1 \Downarrow n_1, M_2}{\sigma, M \vdash e_1 \texttt{ * } e_2 \Downarrow n_1 \times n_2, M_2}$$

[Div] $$\frac{\sigma, M \vdash e_2 \Downarrow n_2, M_1 \qquad \sigma, M_1 \vdash e_1 \Downarrow n_1, M_2}{\sigma, M \vdash e_1 \texttt{ / } e_2 \Downarrow n_1/n_2, M_2}$$

[EqT] $$\frac{\sigma, M \vdash e_2 \Downarrow v_2, M_1 \qquad \sigma, M_1 \vdash e_1 \Downarrow v_1, M_2 \qquad v_1 = v_2}{\sigma, M \vdash e_1 \texttt{ = } e_2 \Downarrow \mathit{true}, M_2}$$

[EqF] $$\frac{\sigma, M \vdash e_2 \Downarrow v_2, M_1 \qquad \sigma, M_1 \vdash e_1 \Downarrow v_1, M_2 \qquad v_1 \neq v_2}{\sigma, M \vdash e_1 \texttt{ = } e_2 \Downarrow \mathit{false}, M_2}$$

[Less] $$\frac{\sigma, M \vdash e_2 \Downarrow n_2, M_1 \qquad \sigma, M_1 \vdash e_1 \Downarrow n_1, M_2}{\sigma, M \vdash e_1 \texttt{ < } e_2 \Downarrow n_1 < n_2, M_2}$$

[Not] $$\frac{\sigma, M \vdash e \Downarrow b, M_1}{\sigma, M \vdash \texttt{not } e \Downarrow \mathit{not}\, b, M_1}$$

[Assign] $$\frac{\sigma, M \vdash e \Downarrow v, M_1}{\sigma, M \vdash x := e \Downarrow v, M_1\{\sigma(x) \mapsto v\}}$$

[Seq] $$\frac{\sigma, M \vdash e_1 \Downarrow v_1, M_1 \qquad \sigma, M_1 \vdash e_2 \Downarrow v_2, M_2}{\sigma, M \vdash e_1 \texttt{ ; } e_2 \Downarrow v_2, M_2}$$

[IfT] $$\frac{\sigma, M \vdash e \Downarrow \mathit{true}, M_1 \qquad \sigma, M_1 \vdash e_1 \Downarrow v, M_2}{\sigma, M \vdash \texttt{if } e \texttt{ then } e_1 \texttt{ else } e_2 \Downarrow v, M_2}$$

[IfF] $$\frac{\sigma, M \vdash e \Downarrow \mathit{false}, M_1 \qquad \sigma, M_1 \vdash e_2 \Downarrow v, M_2}{\sigma, M \vdash \texttt{if } e \texttt{ then } e_1 \texttt{ else } e_2 \Downarrow v, M_2}$$

[WhileT] $$\frac{\sigma, M \vdash e \Downarrow \mathit{true}, M_1 \quad \sigma, M_1 \vdash e_1 \Downarrow v_1, M_2 \qquad \sigma, M_2 \vdash \texttt{while } e \texttt{ do } e_1 \Downarrow v_2, M_3}{\sigma, M \vdash \texttt{while } e \texttt{ do } e_1 \Downarrow v_2, M_3}$$

[WhileF] $$\frac{\sigma, M \vdash e \Downarrow \mathit{false}, M_1}{\sigma, M \vdash \texttt{while } e \texttt{ do } e_1 \Downarrow \mathit{false}, M_1}$$

[Read] $$\frac{}{\sigma, M \vdash \texttt{read } x \Downarrow n, M\{\sigma(x) \mapsto n\}}$$

[Write] $$\frac{\sigma, M \vdash e \Downarrow n, M_1}{\sigma, M \vdash \texttt{write } e \Downarrow n, M_1}$$

[Let] $$\frac{\sigma, M \vdash e_1 \Downarrow v_1, M_1 \quad \ell \notin \mathit{dom}\, M_1 \qquad \sigma\{x \mapsto \ell\}, M_1\{\ell \mapsto v_1\} \vdash e_2 \Downarrow v_2, M_2}{\sigma, M \vdash \texttt{let } x := e_1 \texttt{ in } e_2 \Downarrow v_2, M_2}$$

[LetProc]
$$\frac{x_1 \neq x_2 \qquad \sigma\{f \mapsto \langle [x_1, x_2], e, \sigma \rangle\}, M \vdash e_1 \Downarrow v, M_1}{\sigma, M \vdash \texttt{let proc}\ f\ \texttt{(}\ x_1\texttt{,}x_2\ \texttt{)} = e\ \texttt{in}\ e_1 \Downarrow v, M_1}$$

[CallV]
$$\frac{\begin{array}{c}\sigma(f) = \langle [x_1, x_2], e', \sigma' \rangle \stackrel{\text{let}}{=} p \\ \sigma, M \vdash e_1 \Downarrow v_1, M_1 \qquad \sigma, M_1 \vdash e_2 \Downarrow v_2, M_2 \qquad l_1, l_2 \notin dom\, M_2 \\ \sigma'\{x_1 \mapsto l_1\}\{x_2 \mapsto l_2\}\{f \mapsto p\}, M_2\{l_1 \mapsto v_1\}\{l_2 \mapsto v_2\} \vdash e' \Downarrow v', M'\end{array}}{\sigma, M \vdash f\ \texttt{(}\ e_1\texttt{,}e_2\ \texttt{)} \Downarrow v', M'}$$

[CallR]
$$\frac{\begin{array}{c}\sigma(f) = \langle [x_1, x_2], e', \sigma' \rangle \stackrel{\text{let}}{=} p \\ \sigma, M \vdash e \Downarrow r, M_1 \\ \sigma'\{x_1 \mapsto \sigma(x)\}\{x_2 \mapsto r(y)\}\{f \mapsto p\}, M_1 \vdash e' \Downarrow v', M'\end{array}}{\sigma, M \vdash f\ \texttt{<}\ x\texttt{,}e.y\ \texttt{>} \Downarrow v', M'}$$

[Record]
$$\frac{x_1 \neq x_2 \qquad \sigma, M \vdash e_1 \Downarrow v_1, M_1 \qquad \sigma, M_1 \vdash e_2 \Downarrow v_2, M_2 \qquad l_1, l_2 \notin dom\, M_2}{\sigma, M \vdash \texttt{\{}\ x_1\ \texttt{:=}\ e_1\texttt{,}\ x_2\ \texttt{:=}\ e_2\ \texttt{\}} \Downarrow \texttt{\{}\ x_1 \mapsto l_1,\ x_2 \mapsto l_2\ \texttt{\}}, M_2\{l_1 \mapsto v_1\}\{l_2 \mapsto v_2\}}$$

[Field]
$$\frac{\sigma, M \vdash e \Downarrow r, M_1}{\sigma, M \vdash e\texttt{.}x \Downarrow M_1(r(x)), M_1}$$

[FieldAssign]
$$\frac{\sigma, M \vdash e_1 \Downarrow r, M_1 \qquad \sigma, M_1 \vdash e_2 \Downarrow v, M_2}{\sigma, M \vdash e_1\texttt{.}x\ \texttt{:=}\ e_2 \Downarrow v, M_2\{r(x) \mapsto v\}}$$