퀴즈 3

SNU 4541.664A

10/26(월) 수업시간

이름: 학번:

Exercise 1 요약해석 디자인의 올바름 증명을 완성하라. (강의슬라이드 7, 16-17 쪽 (명령형 언어 요약해석)

$$
\begin{array}{rcl}
C & \to & \texttt{skip} \mid x \texttt{ := } E \mid C \texttt{ ; } C \\
& \mid & \texttt{if } B\ C\ C \\
& \mid & \texttt{while } B\ C \\
E & \to & n \quad (n \in \mathbb{Z}) \mid x \\
& \mid & E \texttt{ + } E \mid B \quad \text{(boolean expr)}
\end{array}
$$

모듬의미 공간은

$$
\begin{array}{rcl}
\underline{C} & \in & 2^{Memory} \to 2^{Memory} \\
\underline{E} & \in & 2^{Memory} \to 2^{Value} \\
\underline{B} & \in & 2^{Memory} \to 2^{Memory} \\
Memory & = & Loc \xrightarrow{\text{fin}} Value \\
Value & = & \mathbb{Z} + \mathbb{B} \\
Loc & = & Var \\
\mathbb{B} & = & \{T, F\}
\end{array}
$$

$$
m \in Memory \qquad M \in 2^{Memory}
$$

$$
\begin{array}{rcl}
\underline{\texttt{skip}}\ M & = & M \\
\underline{x \texttt{ := } E}\ M & = & \{m\{x \mapsto v\} \mid m \in M, v \in \underline{E}\ M\} \\
\underline{C_1 \texttt{ ; } C_2}\ M & = & \underline{C_2}\ (\underline{C_1}\ M) \\
\underline{\texttt{if } B\ C_1\ C_2}\ M & = & \underline{C_1}\ (\underline{B}\ M)\ \cup\ \underline{C_2}\ (\underline{\neg B}\ M) \\
\underline{\texttt{while } B\ C}\ M & = & \underline{\neg B}\ (lfp\lambda X.M \cup \underline{C}\ (\underline{B}\ X)) \\
\underline{n}\ M & = & \{n\} \\
\underline{x}\ M & = & \{m\ x \mid m \in M\} \\
\underline{E_1 \texttt{ + } E_2}\ M & = & \{v_1 + v_2 \mid v_1 \in \underline{E_1}\ M, v_2 \in \underline{E_2}\ M\} \\
\underline{B}\ M & = & \{m \mid \underline{B}\ m = T, m \in M\}
\end{array}
$$

요약의미 공간은

$$\begin{array}{rcl} \underline{C}^{\#} & \in & Memory^{\#} \to Memory^{\#} \\ \underline{E}^{\#} & \in & Memory^{\#} \to Value^{\#} \\ \underline{B}^{\#} & \in & Memory^{\#} \to Memory^{\#} \end{array}$$

갈로아 연결 된 요약공간

$$2^{Memory} \underset{\alpha_1}{\overset{\gamma_1}{\leftrightarrows}} Memory^{\#} \qquad 2^{Value} \underset{\alpha_2}{\overset{\gamma_2}{\leftrightarrows}} Value^{\#}$$

$$\begin{array}{rcl} \underline{\texttt{skip}}^{\#}\ m^{\#} & = & m^{\#} \\ \underline{x\ \texttt{:=}\ E}^{\#}\ m^{\#} & = & m^{\#}\{x \mapsto^{\#} \underline{E}^{\#}\ m^{\#}\} \\ \underline{C_1\ ;\ C_2}^{\#}\ m^{\#} & = & \underline{C_2}^{\#}\ (\underline{C_1}^{\#}\ m^{\#}) \\ \underline{\texttt{if}\ B\ C_1\ C_2}^{\#}\ m^{\#} & = & \underline{C_1}^{\#}\ (\underline{B}^{\#}\ m^{\#}) \cup^{\#} \underline{C_1}^{\#}\ (\underline{\neg B}^{\#}\ m^{\#}) \\ \underline{\texttt{while}\ B\ C}^{\#}\ m^{\#} & = & \underline{\neg B}^{\#}\ (lfp \lambda x^{\#}.m^{\#} \cup^{\#} \underline{C}^{\#}\ (\underline{B}^{\#}\ x^{\#})) \\ \underline{n}^{\#}\ m^{\#} & = & \alpha_2\{n\} \\ \underline{x}^{\#}\ m^{\#} & = & m^{\#}\ at^{\#}\ x \\ \underline{E_1\ \texttt{+}\ E_2}^{\#}\ m^{\#} & = & (\underline{E_1}^{\#}\ m^{\#}) +^{\#} (\underline{E_2}^{\#}\ m^{\#}) \end{array}$$

여기서 $+^{\#}$, $\cdot\{x \mapsto^{\#} \cdot\}$, $at^{\#}$, $\cup^{\#}$, $\underline{B}^{\#}$, $\underline{\neg B}^{\#}$는 해당 연산들을 안전하게 요약한 것들이라고 가정하자.

요약의미의 올바름은:

Theorem[Correctness]

$$\forall C : \underline{C} \circ \gamma_1 \sqsubseteq \gamma_2 \circ \underline{C}^{\#}.$$

Proof.